

First Order Space Time Autoregressive Stationary Model on Petroleum Data

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Abstract. First order Space-Time Autoregressive model is one of the models which involves location and time. STAR(1;1) model stationary can be used to forecast future observation at a location based on one previous time of its own location and the spatial neighborhood. STAR(1;1) model on petroleum productivity data in Balongan, Indramayu, West Java with eigenvalue less than 1. It indicates that STAR (1;1) model on petroleum productivity data in Balongan, Indramayu, West Java meets the stationary requirement.

Keywords : autoregressive, petroleum data, STAR, first order

1. Introduction

Space-Time Autoregressive (STAR) model is the development of a univariate time series model into a multivariate time series model [1-4]. The STAR(1;1) model states that the current time observation at a particular location is influenced by the observation of previous time at the location and its spatial neighborhood which are a part of the same group [13]. For the simplicity of the model, the study will be focused on time lag 1 and spatial lag 1. A simple, stationary time series model is first order Autoregressive model [5-11] : AR(1). AR(1) model developed into Vector Autoregressive(1) model: VAR(1), first order Space Time Autoregressive(1) model: STAR(1;1)[2]. Petroleum production in Balongan, Indramayu, West Java is a time series problem, which can be modeled by AR(1) model, VAR(1) model, and STAR(1;1) model. [12] Providing procedures in the form of; identification, parameter estimation, and diagnostic examination.

2. Methods used

Applying STAR (1;1) stationary model on petroleum data in Balongan, Indramayu, West Java. The STAR(1;1) model states that the current time observation at a particular location is influenced by the observation of previous time at the location and its spatial neighborhood which are a part of the same research group, thus STAR(1;1) model is applied on petroleum data, specifically on well-1 and well-3 petroleum drilling data in Balongan, Indramayu, West Java. For the simplicity of the model, the study will be focused on time lag 1 and spatial lag 1 on STAR(1;1) model at several lag locations. And the STAR(1;1) model is examined or assessed. After the assessment phase, model validation is conducted through *error*. If the model determined as adequate, then it can be used to forecast future observation on petroleum data in Balongan, Indramayu, Jawa Barat.

3. Results and Discussion

Bivariate VAR(1) model is said to be first order stationary, if the mean and covariance are not dependent on t :

$$\begin{aligned} E[z(t)] &= \mu \\ \text{Cov}[z(t), z(s)] &= E[z(t) z(s)'] = E[z(0)z(s-t)'] \\ &= C(0, s-t) = C(t-s) = C(s-t) \end{aligned} \quad (1)$$

From equation (1) covariance matrix C only depends on time shift or time lag $(t-s)$ [5] in [3]

The theorem of VAR(p) model stationary requirement with $u = 1, 2, \dots, N$ variate, presented [5] in [3] as follows:

If x_u equation solution

$$|x_u \mathbf{I} - \sum \Phi(j) x_u| = 0$$

located in a unit circle ($|x_u| < 1$), then the VARMA(p, q) process is stationary.

Next [6] provides Hannan stationary requirement equivalence above for VAR(1) model, as follows:

If $\forall B \in \mathbb{C}$, $|B| > 1$ applies $|\mathbf{I} - \Phi B| = 0$, then VAR(1) is stationary. In other words, the B roots of $|\mathbf{I} - \Phi B| = 0$ are outside the unit circle.

STAR(1;1) model by [4] in [3] is stated:

$$\mathbf{z}(t) = \mathbf{z}(t-1) + \mathbf{W}\mathbf{z}(t) + \mathbf{e}(t) \quad (2)$$

$$\mathbf{W} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \text{ equal weight}$$

STAR(1;1) model equation for 2 locations can be presented as follows:

$$\begin{bmatrix} z_1(t) \\ z_2(t) \end{bmatrix} = \phi_{01} \begin{bmatrix} z_1(t-1) \\ z_2(t-1) \end{bmatrix} + \phi_{11} \begin{bmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \end{bmatrix} \begin{bmatrix} z_1(t-1) \\ z_2(t-1) \end{bmatrix} + \begin{bmatrix} e_1(t) \\ e_2(t) \end{bmatrix} \quad (3)$$

Or

$$\begin{bmatrix} z_1(t) \\ z_2(t) \end{bmatrix} = \phi_{01} \begin{bmatrix} z_1(t-1) \\ z_2(t-1) \end{bmatrix} + \phi_{11} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} z_1(t-1) \\ z_2(t-1) \end{bmatrix} + \begin{bmatrix} e_1(t) \\ e_2(t) \end{bmatrix}$$

Equation (1) can be stated in a form:

$$\begin{aligned} \mathbf{z}(t) &= [\phi_{01} \mathbf{I} + \phi_{11} \mathbf{W}] \mathbf{z}(t-1) + \mathbf{e}(t) \\ \mathbf{z}(t) &= \mathbf{z}(t-1) + \mathbf{e}(t) \end{aligned} \quad (4)$$

$$\mathbf{z}(t) = [\phi_{01} \mathbf{I} + \phi_{11} \mathbf{W}] \mathbf{z}(t-1) + \mathbf{e}(t)$$

with

$$\Phi = [\phi_{01} \mathbf{I} + \phi_{11} \mathbf{W}]$$

Next, validation model is conducted through error. If the model determined as adequate, then it can be used to forecast future observation by using linear equation model [17]:

$$\mathbf{y} = \mathbf{X} \boldsymbol{\beta} + \mathbf{e}(t), \mathbf{e}(t) \stackrel{iid}{\sim} N(\mathbf{0}, \sigma^2) \quad (5)$$

STAR(1;1) model can be written

$$\mathbf{z}(t) = [\mathbf{z}(t-1) \quad \mathbf{W}\mathbf{z}(t-1)] \begin{bmatrix} \phi_{01} \\ \phi_{11} \end{bmatrix} + \mathbf{e}(t) \quad (6)$$

and

$$\mathbf{y} = \mathbf{X} \boldsymbol{\beta} + \mathbf{e}(t)$$

Thus, STAR (1;1) model parameter estimation is obtained by using the smallest square method

$$\hat{\beta} = \begin{pmatrix} \hat{\phi}_{01} \\ \hat{\phi}_{11} \end{pmatrix} = (X' X)^{-1} X' \hat{y}. \quad (7)$$

with

$$X = [z(t-1) \quad Wz(t-1)]$$

Matrix X above is a 2x2 size matrix contains:

$$X = \begin{bmatrix} z_1(t-1) & 0 \\ z_2(t-1) & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} z_1(t-1) \\ z_2(t-1) \end{bmatrix} \quad (8)$$

Theorem: STAR(1;1) is said to be stationary, if it meets the following condition;

$$|\phi_{01}| + |\phi_{11}| < 1 \quad (9)$$

[4] in [2] has proven the stationary requirement as follows:

$$|x_u I - \phi_{10} I - \phi_{11} W| = 0 \quad (10)$$

or

$$|-\phi_{11} W - (\phi_{10} - x_u) I| = 0$$

with $\lambda(x_u) = \phi_{10} - x_u$

and $\lambda(x_u)$ is Eigenvalue $A = -\phi_{11} W$

Each Eigenvalue A located in one of closed unit circles, stated by

$$|x - a_{ij}| \leq \sum_{j \neq 1} |a_{ij}|$$

For STAR (1;1) model case, the application of calculus theorem conducted by taking $A = -\phi W$

and

$$\sum_{j=1} |a_{ij}| = \sum_{j=1} w_{ij} |\phi_{11}| x I = |\phi_{11}|$$

$$w_{ij} = 0$$

$$\sum_{j=1}^N w_{ij} = 1$$

thus

$$a_{ij} = \phi_{11}$$

can be written as :

$$|\lambda - 0| = |\phi_{11}|$$

And because all circles are the same, thus all Eigenvalues need to meet:

$$|\lambda| \leq |\phi_{11}|$$

For STAR(1;1) model $\lambda(x_u) = \phi_{10} - x_u$

therefore

$$|\phi_{10} - x_u| \leq |\phi_{11}|$$

(11)

equation (2) is the same as follows

$$|\phi_{10} - x_u| \leq |\phi_{11}|$$

and

$$x_u - \phi_{10} \leq |\phi_{11}|$$

thus

$$\phi_{10} - |\phi_{11}| \leq x_u \leq \phi_{10} + |\phi_{11}|$$

because

$$|x_u| < 1 \text{ or } -1 < u < 1,$$

thus

$$-1 < \phi_{10} - |\phi_{11}| \leq x_u \leq \phi_{10} + |\phi_{11}| < 1$$

or

$$\phi_{10} - |\phi_{11}| > -1$$

and

$$\phi_{10} + |\phi_{11}| < 1$$

Both conditions combined, so that the STAR (1;1) model will be stationary with

(ϕ_{10}, ϕ_{11}) parameter meets

$$|\phi_{10}| + |\phi_{11}| < 1$$

(12)

STAR(1;1) model on two petroleum wells production data in Balongan Indramayu West Java

Table Petroleum Data

	S1	S3		S1	S3		S1	S3
1.	635	737	21.	262	506	41.	527	486
2.	534	672	22.	333	454	42.	478	426
3.	598	717	23.	288	405	43.	496	462
4.	541	673	24.	321	567	44.	610	513
5.	489	890	25.	539	518	45.	620	457
6.	455	751	26.	446	395	46.	657	476
7.	503	793	27.	374	361	47.	573	432
8.	573	662	28.	420	395	48.	611	337
9.	571	518	29.	501	353	49.	528	383
10.	534	606	30.	520	318	50.	484	398
11.	414	557	31.	512	352	51.	606	387
12.	575	551	32.	471	342	52.	586	403
13.	635	518	33.	464	327	53.	520	421
14.	443	694	34.	505	328	54.	568	330
15.	445	654	35.	577	376	55.	592	347
16.	448	504	36.	428	306	56.	610	402
17.	450	629	37.	404	332	57.	646	409
18.	468	508	38.	392	386	58.	702	432
19.	501	380	39.	494	503	59.	714	388
20.	443	444	40.	533	542	60.	849	306

> Summary

S1 S3

Min.:262.0 Min.:306.0
 1st Qu.:449.5 1st Qu.:382.2
 Median:516.0 Median:438.0
 Mean:516.9 Mean:478.6
 3rd Qu.:579.2 3rd Qu.:544.2
 Max.:849.0 Max.:890.0

> Cor

S1 S3
S1 1.00000000 -0.06468568
S3 -0.06468568 1.00000000

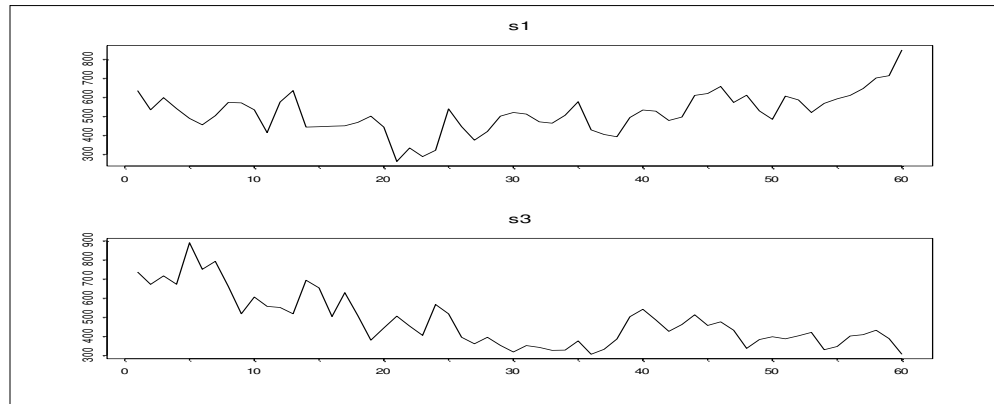


Image 1: graphs graphs of Petroleum Wells Production data

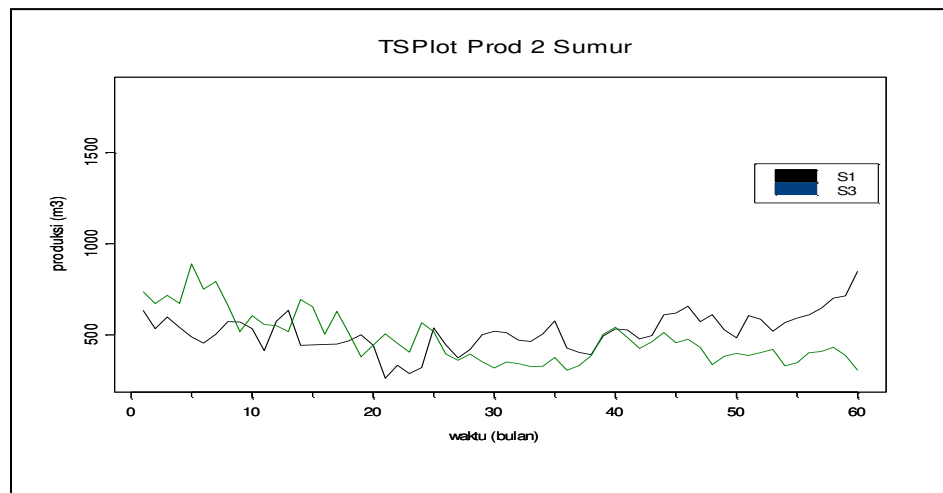


Image 2: Combined graphs of 2 Petroleum Wells Production data

The stationary requirement for STAR(1;1) model is fulfilled, because STAR (1;1) model parameter estimation which is ϕ_{01} and ϕ_{11} values as follows:

$$\phi_{01} = 0,91697$$

$$\phi_{11} = 0,07295$$

Implying stationary requirement is fulfilled as follows

$$|\phi_{01}| + |\phi_{11}| < 1$$

This value indicates that STAR (1;1) model on petroleum production data is stationary. Those values obtained by using Microsoft Excel.

STAR model (*Time Series*) (Excel 2003)

matrix phi

$$\phi_{01} = 0,91697$$

$$\phi_{11} = 0,072946$$

Stationary: 0,989916

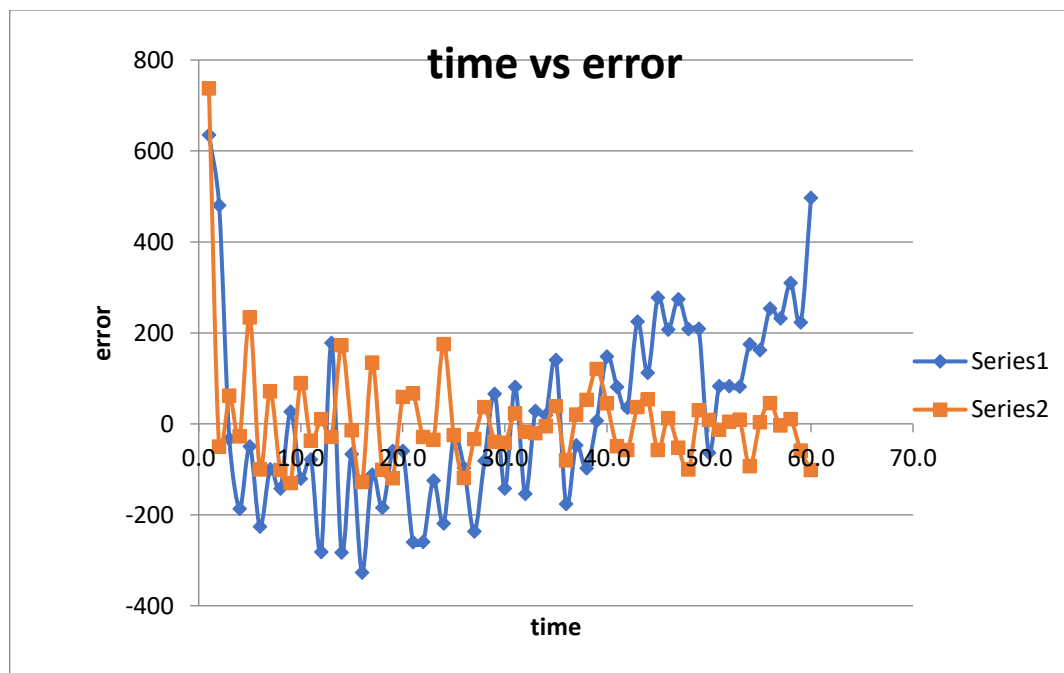


Image 3: Error graph of 2 petroleum wells

4. Conclusion

STAR(1;1) model on two petroleum wells data is stationary and meets the requirement.

$$|\phi_{01}| + |\phi_{11}| < 1$$

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